Mathematical Arguments
Getting to know you

Consider the following mathematical statement:

When you add any two consecutive numbers, the answer is always odd.

Think

1) Is this statement (claim) true?
2) What's your argument to show that it is or is not true?

Pair – Share (if working with a group)

Share your arguments with another person for why the statement is or is not true. What's similar about your arguments? What's different?

Bonus: *With time, consider* How might one of your students respond to the above "Think" questions?

Two students, Micah and Angel, explained why the following statement was true.
When you add any two consecutive numbers, the answer is always odd.

1. Consider each of their responses:

Micah's Response

5 and 6 are consecutive numbers, and 5 + 6 = 11 and 11 is an odd number.

12 and 13 are consecutive numbers, and 12 + 13 = 25 and 25 is an odd number.

1240 and 1241 are consecutive numbers, and 1240 +1241 = 2481 and 2481 is an odd number.

That's how I know that no matter what two consecutive numbers you add, the answer will always be an odd number.

Angel's Response

Consecutive numbers go even, odd, even, odd, and so on. So if you take any two consecutive numbers, you will always get one even and one odd number.

And we know that when you add any even number with any odd number the answer is always odd.

That's how I know that no matter what two consecutive numbers you add, the answer will always be an odd number.

These are not direct responses, but based on actual student responses.[1]

a. Is the response mathematically accurate?
b. Does the response justify the statement? (Does the response offer a mathematical argument that demonstrates the claim to be true?) Explain.

Micah's Response	Angel's Response

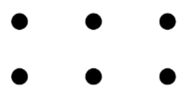
[1] Original problem from Bieda, K, Holden, C., & Knuth, E. (2006). Does proof prove?: Students' emerging beliefs about generality and proof in middle school. *Proceedings of the 28th Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education. Mérida, Mexico: Universidad Pedagógica Nacional* (pp 395 – 402).

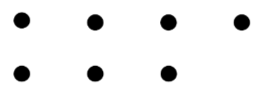
And consider a third response…

Roland's Response

The answer is always odd.

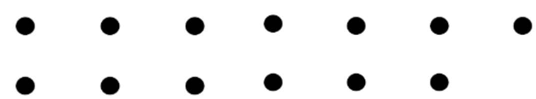
A number + The next number =





An odd number

There's always one left over when you put them together, so it's odd.



Roland's Response
a. Is the response mathematically accurate?
b. Does the response justify the statement? (Does the response offer a mathematical argument that demonstrates the claim to be true?) Explain.

Summary notes on the different approaches, different arguments

- All the 'math' in these responses is **accurate**.

- Each student has taken a **different approach** in formulating his or her argument.

  Example-based (Micah)

  Narrative (Angel)

  Pictorial (Roland) (also can be considered model-based)

  Symbolic (Algebraic) (example shown here)
    Consecutive numbers can be represented as n and n+1.
    Adding these, we get n+(n+1)=2n+1.
    2n+1 is odd because there's one left over when you divide by 2.

  Micah's approach will not produce a valid argument for *<u>this</u> claim*. A set of examples cannot show a general claim to be true. Each of the other approaches can be used to show this claim true, possibly with some revision.

- Each approach shows an understanding of **consecutive numbers**.

- The approaches vary in how they use the idea of **even and odd** numbers. Note, however, that to generate an argument for this claim, one has to consider and use the ideas of even and odd numbers.

- The level of **generality** of these student arguments varies. Micah's is not general. Angel's is general. Roland's visual is not obviously general, but we can see how one might describe the pictures further, or redraw them, to be more general. The symbolic argument above is general.